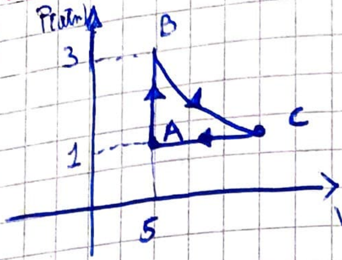
Esercizio 1

$pV = cost$

AB → ISOCORA
BC → ISOTERMA
AC → ISOBARA

$V_A = V_B = 5\,l$
$p_A = p_C = 1\,atm$
$T_A = 300\,K$
$p_B = 3\,atm$

$R = 8{,}315 \frac{J}{mol\,K}$



n → costant (sistema chiuso), sono conservate durante la trasformazione
↳ Possiamo scegliere un punto qualsiasi.

$$n = \frac{p_A V_A}{R T_A} = 0{,}203\,mol \qquad T_C = T_B = \frac{p_B V_B}{nR} = 900\,K$$

$$V_C = \frac{n R T_C}{p_C} \Rightarrow 15\,l$$

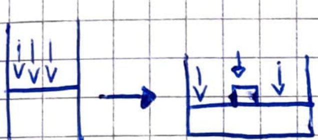
$$L_{AB} = 0 \to \int_A^B p\,dV \to 0 \Rightarrow \boxed{L_{AB} = 0}$$

$$L_{BC} = \int_{V_B}^{V_C} p\,dV = \int_{V_B}^{V_C} \frac{nRT_B}{V}\,dV = nRT_B \int_{V_B}^{V_C} \frac{1}{V}\,dV = nRT_B \cdot \ln\left(\frac{V_C}{V_B}\right) = 1{,}67 \cdot 10^3$$

$$L_{CA} = p_a (V_A - V_C) = -1{,}01 \cdot 10^3\,J$$

$$\boxed{L_{TOT} = L_{AB} + L_{BC} + L_{CA}}$$

Esercizio 2



$r = 14\,cm$ $\quad c_v = \frac{3}{2}R$
$n = 1{,}5\,mol$
$T = 25°C$ $\quad p_A = 1\,atm$ $\quad m = 30\,Kg$

$Q = 0$ → trasformazione adiabatica

$\Delta U = Q - L$
- ΔU: positivo se T aumenta, negativo se T diminuisce
- L: positivo se fatto dal gas, negativo se fatto sul gas
- Q: positivo se assorbito dal gas, negativo se ceduto dal gas

$$\Delta U = n c_v \Delta T \to n \frac{3}{2} R$$

$$-\Delta U = +L$$

$$\left(p_a + \frac{mg}{S}\right)(V_2 - V_1) = -n c_v \Delta T$$

$$V_2 = \frac{nRT_2}{p_2} = \frac{nRT_2}{p_a + \frac{mg}{S}}$$

$$V_1 = \frac{nRT_1}{p_a}$$

$$\left(p_a + \frac{mg}{S}\right)\left(\frac{nRT_2}{p_a + \frac{mg}{S}} - \frac{nRT_1}{p_a}\right) = -\frac{3}{2}Rn(T_2 - T_1)$$

$$nRT_2 - \left(p_a + \frac{mg}{S}\right)\left(\frac{nRT_1}{p_a}\right) = -\frac{3}{2}Rn(T_2 - T_1)$$

$$T_2 = T_1 + \frac{2 T_1 mg}{5 p_a \cdot S} \Rightarrow 303{,}6\,K$$

Esercizio 3

$Q = 0 \qquad L = -\Delta U \to L = -n c_v \Delta T$

$T_1 = 300\,K \quad V_1 = 1\,m^3 \quad V_2 = 2\,m^3$

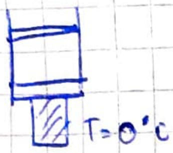
$$\gamma = \frac{c_p}{c_v} = \frac{5}{3} \qquad c_p = c_v + R \qquad c_p = \frac{5}{2}R \qquad c_v = \frac{1}{n}\left(\frac{dU}{dT}\right)_V = \frac{3}{2}R$$

$$pV^{\gamma} = cost \qquad TV^{\gamma-1} = cost$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \to T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \simeq 189\,K$$

$$L = -n\frac{3}{2}R(T_2 - T_1) = 1384\,J$$

Esercizio 4



$m = 0{,}1$ $\quad T = 0°C = 273{,}16\ K$

$V_1 = 1\ m^3$

$n = 10\ mol$ $\quad V_f < V_i$

ISOTERMA → $T = cost$ → $\Delta U = 0$

$Q = L$ $\quad L = \int_{V_i}^{V_f} p\,dV = nRT \ln\left(\frac{V_f}{V_i}\right)$ $\quad Q = -m_g \lambda_g$ (fondere il ghiaccio)

$$nRT \ln\left(\frac{V_f}{V_i}\right) = -m_g \lambda_g$$

$$V_f = V_i\, e^{-\frac{m\lambda_g}{nRT}} = \boxed{0{,}23\ m^3}$$

Esercizio 5

$n_1 = 3\ mol$ He $\quad n_2 = 2\ mol$ N_2

$T_1 = 300\ K$ $\quad T_2 = 270\ K$

$c_{V1} = \frac{3}{2}R$ $\quad c_{V2} = \frac{5}{2}R$

Non viene compiuto lavoro → $L = 0$

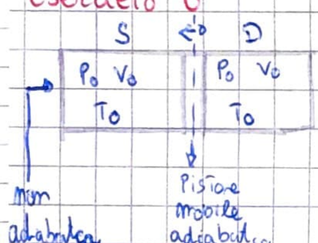
Non viene scambiato calore → $Q = 0$ → pareti adiabatiche

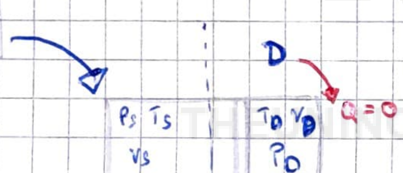
$$\Delta U_{TOT} = \underbrace{\Delta U_1}_{Q_{ced}} + \underbrace{\Delta U_2}_{Q_{acq}} = 0$$

$$n_1 c_{V1} (T_e - T_1) + n_2 c_{V2} (T_e - T_2) = 0$$

$$T_{eq} = \frac{n_1 c_{V1} T_1 + n_2 c_{V2} T_2}{n_1 c_{V1} + n_2 c_{V2}} = 284\ K$$

Esercizio 6





$V_0 = 100\ l$ $\quad c_V = \frac{3}{2}R$ $\quad p_0 = 1\ atm$ $\quad T_0 = 10°C$

$p_D = 5\ atm$

D → $p_0 V_0^\gamma = p_D V_D^\gamma$

$$p_0 \left(\frac{nRT_0}{p_0}\right)^\gamma = p_D \cdot \left(\frac{nRT_D}{p_D}\right)^\gamma$$

$$p_0^{1-\gamma} T_0^\gamma = p_D^{1-\gamma} T_D^\gamma$$

$$T_D = \left(\frac{p_0}{p_D}\right)^{\frac{1-\gamma}{\gamma}} \cdot T_0 \Rightarrow T_D = 538{,}7\ K$$

DESTRA

$\Delta U = -W$ → $W = -\Delta U$ → $-nc_V(T_D - T_0)$ (inverto perché c'è il −)

$n = \frac{p_0 V_0}{R T_0}$ ↓ il numero di mol è sempre uguale

$$= \frac{p_0 V_0}{R T_0} \cdot \frac{3}{2} R \cdot (T_0 - T_D)$$

$$\Rightarrow \boxed{L = -1{,}37 \cdot 10^4\ J}$$

$p_D = p_S$ $\quad V_S = 2V_0 - V_D$ → $V_S = 2V_0 - \left(\frac{p_0}{p_D}\right)^{\frac{1}{\gamma}} V_0 = V_0\left[2 - \left(\frac{p_0}{p_D}\right)^{\frac{1}{\gamma}}\right] \Rightarrow V_S = V_0\left[2 - \left(\frac{p_0}{p_D}\right)^{\frac{1}{\gamma}}\right]$

$p_0 V_0^\gamma = p_D V_D^\gamma$ → $V_D = \left(\frac{p_0}{p_D}\right)^{\frac{1}{\gamma}} \cdot V_0$

$p_S V_S = nRT_S$ → $n = \frac{p_0 V_0}{R T_0} \cdot R \cdot T_S$ $\quad T_S = \frac{p_S}{p_0} \cdot \frac{V_S}{V_0} \cdot T_0 = 2291\ K$

Esercizio 7

$\Delta V = -10^{-3}\, m^3$

$n = 0{,}5\ mol$

$P_f = 2 \cdot 10^5\ Pa$

$C = 5\ J/K$

$\Delta U = Q - L$

$\Delta U = m c_v \Delta T$

$Q = C\,\Delta T \rightarrow C\Delta T^{AMBIENTE} - (C\Delta T)^{GAS} = 0$ ← calore ceduto dal gas all'ambiente esterno

$\Delta U = Q - L \rightarrow m c_v \Delta T = -C\Delta T^{GAS} - P_f \Delta V$

$$\Delta T = \frac{-P_f \Delta V}{m c_v + C} = 17{,}8\ K$$