30/11/2020

lunedì 30 novembre 2020 16:12

CORREZIONE

$$\underbrace{\int e^{2x}\underbrace{\sin 3x}_{F}\,dx}_{INT} = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\underbrace{\cos 3x}_{F}\,dx =$$

$$= \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\left[\frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x\,dx\right]$$

$$INT = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{4}e^{2x}\cos 3x - \frac{9}{4}\,INT$$

$$\frac{13}{\cancel{4}}\int e^{2x}\sin 3x = \frac{e^{2x}}{\cancel{4}}\left(2\sin 3x \cdot 3\cos 3x\right) + c$$

$$\int e^{2x}\sin 3x = \frac{e^{2x}}{13}\left(2\sin 3x - 3\cos 3x\right) + c$$

$$\int x|x-2|\,dx = \begin{cases} \to x>2 & \int x^2 - 2x\,dx = \frac{x^3}{3} - x^2 + c \\ \to x<2 & \int -x^2 + 2x\,dx = -\frac{x^3}{3} + x^2 + c \end{cases}$$

$$\int x|x-2|\,dx = \begin{cases} \frac{x^3}{3} - x^2 + c & x>2 \\ \frac{-x^3}{3} + x^2 + c & x<2 \end{cases}$$

NO!

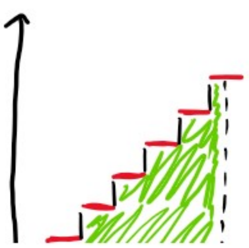
$$F(x) = \frac{|x|^3}{3} \qquad \int x|x-2|\,dx = F(x) + c$$

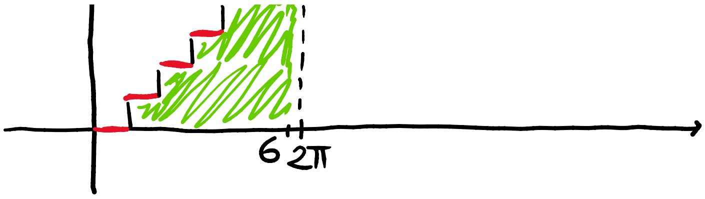
per concludere manca il 2 DEVE ESSERE CONTINUA

$$\int x|x-2|\,dx = \begin{cases} \frac{x^3}{3} - x^2 + c \quad x>2 \xrightarrow{2^+} c_1 - \frac{4}{3} \\ -\frac{x^3}{2} + x^2 + c \quad x<2 \xrightarrow{2^-} c_2 + \frac{4}{3} \qquad c_2 = c_1 - \frac{8}{3} \end{cases}$$

$$\int x|x-2|\,dx = \begin{cases} \frac{x^3}{3} - x^2 + c & x \geq 2 \\ -\frac{x^3}{3} + x^2 - \frac{8}{3} + c & x<2 \end{cases}$$

$$\int_0^{2\pi} \lfloor x \rfloor\,dx = 15 + 6(2\pi - 6) = 12\pi - 19$$





$[a,b]$ limitato, f limitata

INTEGRALI GENERALIZZATI

- $f \in C(a,b]$ $\quad \lim_{x\to a^+} f(x) = \pm\infty$

$$\lim_{\varepsilon\to 0^+} \int_{a+\varepsilon}^{b} f(x)\,dx = \begin{cases} \exists \text{ finito} \to \text{l'integrale converge} \\ \exists \text{ infinito} \to \text{l'integrale diverge} \\ \nexists \to \text{l'integrale è indeterminato} \end{cases}$$

es.

$$\int_0^1 \ln x\,dx = \lim_{\varepsilon\to 0^+} \int_{0+\varepsilon}^1 \ln x\,dx =$$

$$= \lim_{\varepsilon\to 0^+} \left[x \ln x\right]_\varepsilon^1 = \lim_{\varepsilon\to 0^+} (-1-\underbrace{\varepsilon\ln\varepsilon}_{\to 0}+\underbrace{\varepsilon}_{\to 0}) = -1$$

INTEGRALE CONVERGE

es.

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{\varepsilon\to 0^+} \int_\varepsilon^1 \frac{1}{x^2}\,dx = \lim_{\varepsilon\to 0^+} \left[-\frac{1}{x}\right]_\varepsilon^1 = \lim_{\varepsilon\to 0^+} \left(-1+\frac{1}{\varepsilon}\right) =$$

$$= -1$$

se $f(x) \xrightarrow{x\to a^+} \pm\infty$ e $f(x) \xrightarrow{x\to b^-} \pm\infty$ e $f \in C(a,b)$

$$\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x) =$$

$$= \lim_{\varepsilon\to 0^+} \int_{a+\varepsilon}^c f(x) + \lim_{\varepsilon\to 0^+} \int_c^{b-\varepsilon} f(x)$$

- CONVERGE solo se convergono separatamente i due integrali

COMPITO

$$\int \frac{1}{x^\alpha}\,dx = \begin{cases} \ln x + c & \alpha = 1 \\ \dfrac{1}{(1-\alpha)x^{\alpha+1}} & \alpha \neq 1 \end{cases}$$

$$\int_0^1 \frac{1}{x^\alpha}\,dx = \lim_{\varepsilon\to 0^+} \int_0^1 \frac{1}{x^\alpha}\,dx \begin{cases} \to \alpha \geq 1 \text{ diverge} \\ \to \alpha < 1 \text{ converge} \end{cases}$$

DA CONTROLLARE

$\underbrace{\qquad}_{\text{DA CONTROLLARE}} \to \alpha < 1$ converge

CRITERIO del CONFRONTO

$f, g \in \mathcal{C}(a,b]$, f e $g \xrightarrow{x \to a} +\infty$, $0 \leq f(x) \leq g(x)$

se $\int_a^b g(x)dx$ CONVERGE

$\Rightarrow f(x)$ CONVERGE

se $\int_a^b f(x)dx$ DIVERGE

$\Rightarrow g(x)$ DIVERGE

es. $\int_0^1 \sqrt[3]{-\ln x}\, dx = \int_0^{\frac{1}{e}} \sqrt[3]{-\ln x}\, dx + \underbrace{\int_{1/e}^1 \sqrt[3]{-\ln x}\, dx}_{= \text{a un valore}}$

$x < \frac{1}{e}$ $\quad -\ln x > 1$

$\underbrace{\sqrt[3]{-\ln x}}_{f} < \underbrace{-\ln x}_{g}$

se $\int_0^{\frac{1}{e}} -\ln x$ CONVERGE anche $\int_0^{\frac{1}{e}} \sqrt[3]{-\ln x}$ CONVERGE

CRITERIO del CONFRONTO ASINTOTICO

f e $g \in \mathcal{C}(a,b]$, f e $g > 0$, f e $g \xrightarrow{x \to a^+} +\infty$, $f(x) \overset{x \to a}{\sim} g(x)$

$\Rightarrow \int_a^b f(x)$ converge se $\int_a^b g(x)dx$ converge

es.

$\int_0^1 \frac{\sin\sqrt{x}}{x}\, dx$

$x \to 0$ $\quad \frac{\sin\sqrt{x}}{x} \sim \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ $\qquad \alpha = \frac{1}{2} < 1$ CONVERGE

COMPITO

$\gamma \in \mathbb{R}$ $\quad \int_0^{2\pi} (\operatorname{tg} x)^\gamma\, dx$ $\quad$ per quali valori di γ converge?

CRITERIO

Se $f \in \mathcal{C}(a,b]$, f cambia segno ripetutamente per $x \to a^+$

se $\int_a^b |f(x)|$ CONVERGE

$\Rightarrow \int_a^b f(x)\, dx$ CONVERGE se $\int_a^b |f(x)|$ converge

$\Rightarrow \int_a^b f(x)\,dx$ CONVERGE

es.

$$\int_0^1 \frac{1}{\sqrt{x}} \sin \frac{1}{x^2}\,dx$$

$$0 \leq \left| \frac{1}{\sqrt{x}} \sin \frac{1}{x^2} \right| \leq \frac{1}{\sqrt{x}}$$

$\int_0^1 \left| \frac{1}{\sqrt{x}} \sin \frac{1}{x^2} \right| dx$ $\qquad$ $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ CONVERGE $\alpha = \frac{1}{2} < 1$

$\downarrow$

CONERGE e quindi anche il primo integrale CONVERGE

se f e' limitata

$$\int_a^{+\infty} f(x)\,dx$$

$$\lim_{R \to +\infty} \int_a^R f(x)\,dx = \begin{cases} \exists \text{ finito} & \text{CONVERGE} \\ \exists \text{ infinito} & \text{DIVERGE} \\ \nexists & \text{INDETERMINATO} \end{cases}$$

es.

$$\int_2^{+\infty} e^{-x}\,dx = \lim_{R \to +\infty} \underbrace{\int_2^R e^{-x}\,dx}_{\left[-e^{-x}\right]_2^R = e^{-2} - e^{-R}} = \lim_{R \to +\infty} e^{-2} - e^{-R} = e^{-2}$$

$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{R \to +\infty} \underbrace{\int_1^R \frac{1}{x}\,dx}_{\left[\ln|x|\right]_1^R = \ln R} = \lim_{R \to +\infty} \ln R = +\infty$$

COMPITO

$$\int_1^{+\infty} \frac{1}{x^\beta}\,dx = \begin{cases} \beta > 1 & \text{DIVERGE} \\ \beta \leq 1 & \text{CONVERGE} \end{cases}$$

se $f \in C(-\infty, b]$

$$\int_{-\infty}^b f(x)\,dx = \lim_{R \to +\infty} \int_{-R}^b f(x)\,dx$$

se $f \in C\,\mathbb{R}$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{+\infty} f(x)\,dx =$$

se g c ∈ ℝ

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{+\infty} f(x)\,dx =$$

$$= \lim_{R\to+\infty} \int_{-R}^{c} f(x)\,dx + \lim_{R\to+\infty} \int_{c}^{R} f(x)\,dx$$

es.

$$\int_{0}^{+\infty} \cos x\,dx = \lim_{R\to+\infty} \underbrace{\int_{0}^{R} \cos x\,dx}_{[\sin x]_0^R = \sin R} = \lim_{R\to+\infty} \sin R \quad \nexists$$

Vale

- CRITERIO del CONFRONTO

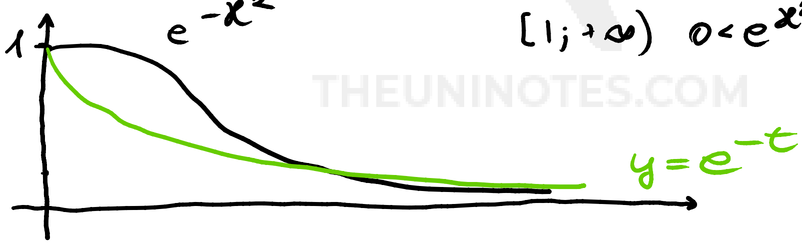
$f, g \in \mathcal{C}[a;+\infty)$ se $0 \le f(x) \le g(x)$

$$\int_{a}^{+\infty} g(x) \text{ CONVERGE} \Rightarrow \int_{a}^{+\infty} f(x) \text{ CONVERGE}$$

$$\int_{a}^{+\infty} f(x) \text{ DIVERGE} \Rightarrow \int_{a}^{+\infty} g(x) \text{ DIVERGE}$$

es.

$$\int_{0}^{+\infty} e^{-x^2}dx = \int_{0}^{1} e^{-x^2}dx + \underbrace{\int_{1}^{+\infty} e^{-x^2}dx}_{[1;+\infty) \quad 0 < e^{-x^2} \le e^{-x}} = \sqrt{\pi}$$



$$\int_{1}^{+\infty} e^{-x}\,dx \quad \text{CONVERGE} \Rightarrow \text{anche l'integrale CONVERGE}$$

- CRITERIO del CONFRONTO ASINTOTICO

f e $g \in \mathcal{C}[a,+\infty)$ f e $g > 0$ $f \overset{x\to+\infty}{\sim} g$

$$\int_{a}^{+\infty} f(x)\,dx \text{ CONVERGE} \Leftrightarrow \int_{a}^{+\infty} g(x) \text{ CONVERGE}$$

es.

$$\int_{3}^{+\infty} \underbrace{\frac{\sqrt{x^3+e^{-x}}}{x^3+\ln x^5}}\,dx \quad \text{CONVERGE}$$

$$\int_3^{+\infty} \underbrace{\frac{\sqrt{x^3+e^{-x}}}{x^3+\ln x^5}}_{f} dx \quad \text{CONVERGE}$$

$$x \to +\infty \quad f(x) \sim \frac{x^{3/2}}{x^3} = \frac{1}{x^{3/2}}$$

$$\beta = \frac{3}{2} > 1 \text{ CONVERGE}$$

COMPITO

$$\int_0^{+\infty} \frac{1-e^{-\sqrt{x}}}{x^{\gamma/3}\sqrt{x^2+8}} \qquad \text{per quali } \gamma \text{ converge?}$$

$$\int_0^c \dots + \int_c^{+\infty} \dots$$

COMPITO

$$\int_a^{+\infty} f(x) \text{ CONVERGE} \qquad \lim_{x\to+\infty} f(x) = 0$$

DOMANDA: c'è qualche implicazione tra le due?