INTEGRALI

Siano $f(x)$ e $F(x)$ funzioni continue su I. $F(x)$ é primitiva di $f(x)$ su I, se F é derivabile su I e se $F'(x) = f(x)$ $\forall x \in I$

$$\int f(x)\,dx = F(x) + c$$

funzione integranda — primitiva

Funzioni elementari

- $\int (3x^2 - 2x + 5)\,dx = \int 3x^2 dx - \int 2x\,dx + \int 5\,dx = 3\int x^2 dx - 2\int x\,dx + 5\int 1\,dx = x^3 - x^2 + 5x + c$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

- $\int \frac{x^2 - 1}{x}\,dx = \int \frac{x^2}{x} - \frac{1}{x}\,dx = \int x - \frac{1}{x}\,dx = \int x\,dx - \int \frac{1}{x}\,dx = \frac{x^2}{2} - \ln|x| + c$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

- $\int e^x + \sin x - \cos 2x\,dx = \int e^x + \int \sin x - \int \cos 2x\,dx = e^x - \cos(x) - \frac{\sin 2x}{2} + c$

$\int e^x dx = e^x + c$ $\quad$ $\int \sin x\,dx = -\cos x + c$

- $\int \frac{1}{1+x^2} + \frac{2}{\cos^2 x}\,dx = \int \frac{1}{1+x^2}\,dx + 2\int \frac{1}{\cos^2 x}\,dx = \operatorname{arctg} x + 2\tan x + c$

- $\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$ $\quad$ $\int \frac{1}{\cos^2 x} = \tan x$

REGOLE DI INTEGRAZIONE

$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + c$ $\quad$ $\int [f(x)]^\alpha \cdot f'(x)\,dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + c$

$\alpha \neq 1$

$\int \frac{1}{x}\,dx = \ln|x| + c \to \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int \sin x\,dx = -\cos x + c \to \int \sin(f(x)) \cdot f'(x)\,dx = -\cos(f(x)) + c$

$\int \cos(x)\,dx = \sin x + c \to \int \cos(f(x)) \cdot f'(x)\,dx = \sin(f(x)) + c$

$\int e^x\,dx = e^x + c \to \int e^{f(x)} f'(x)\,dx = e^{f(x)} + c$

$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c \to \int \frac{f'(x)}{1+(f(x))^2}\,dx = \operatorname{arctg}(f(x)) + c$

Esempi $\int \frac{\ln(x)}{x}\,dx$ $x>0$ $f(x)=\ln(x)$ $f'(x)=\frac{1}{x}$ $\int f(x)\cdot f'(x)\,dx = \frac{(f(x))^2}{2}+c = \frac{(\ln(x))^2}{2}+c$

$\int e^{\sin x}(-\cos x)\,dx = -\int e^{\sin x}(\cos x)\,dx = -\int e^{f(x)}\cdot f'(x)\,dx = -e^{f(x)}+c = -e^{\sin x}+c$

$f(x)=\sin x$
$f'(x)=\cos x$

- $\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int -\frac{f'(x)}{f(x)}\,dx = -\ln|f(x)|+c = -\ln|\cos(x)|+c$ $x\neq\frac{\pi}{2}+k\pi$

$f'(x)=\sin x$
$f(x)=\cos x$

- $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\int \frac{f'(x)}{f(x)}\,dx = \frac{1}{2}\ln|f(x)|+c = \frac{1}{2}\ln(x^2+1)+c$

$f(x)=x^2+1$
$f'(x)=2x$

- $\int \frac{3+x-\operatorname{arctg}x}{x^2+1}\,dx = \int \frac{3}{x^2+1} + \frac{x}{x^2+1} - \frac{\operatorname{arctg}(x)}{x^2+1}\,dx = 3\int \frac{1}{x^2+1}\,dx + \int \frac{x}{x^2+1}\,dx - \int \frac{\operatorname{arctg}(x)}{x^2+1}\,dx$

$f(x)=\operatorname{arctg}(x)$
$f'(x)=\frac{1}{1+x^2}$

$\int \frac{\operatorname{arctg}x}{x^2+1}\,dx = \int f(x)\cdot f'(x)\,dx = \frac{(f(x))^2}{2}+c = \frac{1}{2}(\operatorname{arctg}x)^2+c$

$3\operatorname{arctg}x + \frac{1}{2}\ln(x^2+1) - \frac{1}{2}(\operatorname{arctg}x)^2+c$

- $\int \frac{x-2}{x-3}\,dx = \int \frac{x-2-3+3}{x-3}\,dx = \int \frac{x-3+1}{x-3}\,dx = \int \frac{x-3}{x-3}+\frac{1}{x-3}\,dx \overset{x\neq3}{=} \int 1+\frac{1}{x-3}\,dx$

$= \int 1\,dx + \int \frac{1}{x-3}\,dx = x+\ln|x-3|+c$

INTEGRAZIONE DI FUNZIONI RAZIONALI FRATTE

∂ = grado

$f(x)=\frac{A(x)}{B(x)}$ $A(x)$ e $B(x)$ sono polinomi in x

1° CASO: $\partial A(x) \geq \partial B(x)$ → divido i polinomi.

$A(x) = B(x)\cdot Q(x) + \text{Resto}\ (R(x))$

$\frac{A(x)}{B(x)} = \frac{\cancel{B(x)}\cdot Q(x)}{\cancel{B(x)}} + \frac{R(x)}{B(x)} = Q(x) + \frac{R(x)}{B(x)}$ $\partial R(x) < \partial B(x)$

2° CASO $\partial A(x) < \partial B(x)$

Se $\partial B(x) = 1 \rightarrow \partial A(x) = 0 \rightarrow A(x) =$ costante

$B(x) = px+q$ (grado 1)

$$\int \frac{c}{px+q}\,dx = \frac{c}{p} \cdot \ln|px+q| + k$$

$\partial B(x) = 2 \rightarrow \partial A(x) = 0 \ \vee \ \partial A(x) = 1$

$B(x) = ax^2 + bx + c \quad a \neq 0$ → $A(x) = K$ / → $A(x) = px+q$

$\Delta = b^2 - 4ac$

- Se $\Delta > 0$ 2 soluzioni reali distinte x_1, x_2

$B(x) = a(x-x_1)(x-x_2)$

$$\frac{A(x)}{a(x-x_1)(x-x_2)} = \underbrace{\frac{\alpha}{a(x-x_1)} + \frac{\beta}{(x-x_1)}}$$

$$\int \frac{\alpha}{a(x-x_1)} + \frac{\beta}{(x-x_2)}\,dx = \frac{\alpha}{a}\ln|x-x_1| + \beta\ln|x-x_2| + c$$ sono integrabili

- Se $\Delta = 0$ 2 soluzioni reali e coincidenti $x_1 = x_2$

$B(x) = a(x-x_1)^2$

- Se $A(x) = K$ $\quad \int \frac{K}{a(x-x_1)^2}\,dx = \frac{K}{a}\int (x-x_1)^{-2}\,dx = \frac{K}{a}\frac{(x-x_1)^{-1}}{-1} + c$

- Se $A(x) = px+q$ $\quad \underbrace{\int \frac{px}{a(x-x_1)^2}\,dx} + \int \frac{q}{a(x-x_1)^2}\,dx$

aggiunto il numeratore cercando la derivata di $(x-x_1)^2$ → il numeratore deve essere la derivata del denominatore

- Se $\Delta < 0$ non ci sono soluzioni reali

$B(x) = 1 + (sx+t)^2$

- Se $A(x) = K$ $\quad \int \frac{K}{1+(sx+t)^2}\,dx = \frac{K}{s}\int \frac{s}{1+(sx+t)^2}\,dx = \frac{K}{s}\,\mathrm{arctg}\,(sx+t) + c$

- Se $A(x) = px+q$ $\quad \underbrace{\int \frac{px}{1+(sx+t)^2}\,dx} + \int \frac{q}{1+(sx+t)^2}$

aggiunto il numeratore cercando la derivata di $(sx+t)^2$ → il numeratore deve essere la derivata del denominatore

Esercizi: 1 $\int \frac{1+2x}{2x^2+x-3}\,dx \qquad B(x) = 2x^2+x-3$

$\Delta = 1+24 = 25 > 0$

$B(x) = \frac{-1 \pm 5}{4} = \begin{matrix} 1 \\ -\frac{3}{2} \end{matrix}$

$(2x+3)(x-1)$

$$\int \frac{1+2x}{(2x+3)(x-1)}\,dx$$

$$\frac{1+2x}{(2x+3)(x-1)} = \frac{\alpha}{(2x+3)} + \frac{\beta}{(x-1)} = \frac{\alpha(x-1)+\beta(2x+3)}{(2x+3)(x-1)}$$

$$\alpha x - \alpha + 2\beta x + 3\beta = 1+2x \qquad \begin{cases} \alpha + 2\beta = 2 \\ 3\beta - \alpha = 1 \end{cases} \to \begin{cases} \alpha = 2-2\beta \\ 3\beta - 2 + 2\beta = 1 \end{cases} \to \begin{cases} \alpha = 4/5 \\ \beta = 3/5 \end{cases}$$

$$\frac{1+2x}{(2x+3)(x-1)} = \frac{4/5}{(2x+3)} + \frac{3/5}{(x-1)}$$

$$\int \frac{4/5}{2x+3}\,dx + \int \frac{3/5}{x-1}\,dx = \frac{2}{5}\int \frac{2}{2x+3}\,dx + \frac{3}{5}\ln|x-1| + c$$

$$\frac{2}{5}\ln|2x+3| + \frac{3}{5}\ln|x-1| + c$$

2 $\int \frac{3x-1}{x^2-6x+9}\,dx \qquad B(x) = x^2-6x+9 \qquad B(x) = (x-3)^2$

$\Delta = 0$

$$\int \frac{3x-1}{(x-3)^2}\,dx \qquad \int \frac{3x}{x^2-6x+9} - \frac{1}{x^2-6x+9}\,dx$$

$B(x) = x^2-6x+9$

$B'(x) = 2x-6$

$$=3\int \frac{x}{x^2-6x+9}\,dx = \frac{3}{2}\int \frac{2x}{x^2-6x+9}\,dx = \frac{3}{2}\int \frac{2x-6}{x^2-6x+9} + \frac{6}{x^2-6x+9}\,dx$$

$$\int \frac{3x}{x^2-6x+9} - \frac{1}{(x-3)^2}\,dx = \frac{3}{2}\int \frac{2x-6}{x^2-6x+9}\,dx + \frac{3}{2}\int \frac{6}{(x-3)^2}\,dx - \int \frac{1}{(x-3)^2}\,dx$$

$$= \frac{3}{2}\ln|x^2-6x+9| + 9\int \frac{1}{(x-3)^2}\,dx - \int \frac{1}{(x-3)^2}\,dx \Rightarrow \frac{3}{2}\ln|x^2-6x+9| + 8\int \frac{1}{(x-3)^2}\,dx$$

$$\Rightarrow \frac{3}{2}\ln|x^2-6x+9| + 8\,\frac{(x-3)^{-1}}{-1} + c = \frac{3}{2}\ln(x-3)^2 - \frac{8}{x-3} + c \Rightarrow 3\ln|x-3| - \frac{8}{x-3} + c$$

③ $\int \frac{2x-1}{4x^2+1}\,dx$ $\quad B(x) = 4x^2+1$ $\quad \Delta < 0$ $\quad B(x) = 1+(2x)^2$

moltiplico per 4 in modo da avere $B'(x) = 8x$

$$\frac{1}{4}\int \frac{8x-4}{4x^2+1}\,dx = \frac{1}{4}\int\left(\frac{8x}{4x^2+1} - \frac{4}{4x^2+1}\right)dx$$

$$= \frac{1}{4}\int \frac{8x}{4x^2+1}\,dx - \left(\frac{1}{4}\cdot 4\right)\int \frac{1}{4x^2+1}\,dx$$

$$\frac{1}{4}\ln(4x^2+1) - \int \frac{1}{1+(2x)^2}\,dx$$

$f(x) = 2x$
$f'(x) = 2$

$$\to \frac{1}{2}\int \frac{2}{1+(2x)^2} = \frac{1}{2}\,\text{arctg}\,2x$$

$$\frac{1}{4}\ln(4x^2+1) - \frac{1}{2}\,\text{arctg}\,2x + c$$

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④ $\int \frac{4-x}{x^2-6x+12}\,dx$ $\quad B(x) = x^2-6x+12$ $\quad \Delta < 0$

completo il quadrato

$B(x) = (x^2-6x+9)+3$ → devo arrivare a 12

$= (x-3)^2+3 = 3\left(\frac{(x-3)^2}{3}+1\right)$

$= 3\left[1+\left(\frac{x-3}{\sqrt{3}}\right)^2\right]$

$B'(x) = 2x-6$

moltiplico per -2

$$-\frac{1}{2}\int \frac{-8+2x}{x^2-6x+12} = -\frac{1}{2}\int \frac{2x-6}{x^2-6x+12} - \frac{2}{x^2-6x+12}\,dx$$

$$-\frac{1}{2}\int \frac{2x-6}{x^2-6x+12}\,dx + \left(-\frac{1}{2}\cdot(-2)\right)\cdot\int \frac{1}{x^2-6x+12}\,dx$$

$$-\frac{1}{2}\ln|x^2-6x+12| \cancel{dx} + \int \frac{1}{\left[1+\left(\frac{x-3}{\sqrt{3}}\right)^2\right]3}\,dx$$

$$-\frac{1}{2}\ln|x^2-6x+12| \cancel{dx} + \frac{1}{3}\int \frac{1}{1+\left(\frac{x-3}{\sqrt{3}}\right)^2}\,dx$$

$f(x) = \frac{x-3}{\sqrt{3}}$ $\quad f'(x) = 1/\sqrt{3}$

$$-\frac{1}{2}\ln|x^2-6x+12| \cancel{dx} + \sqrt{3}\,\frac{1}{3}\int \frac{1}{1+\left(\frac{x-3}{\sqrt{3}}\right)^2}\,dx$$

$$\boxed{-\frac{1}{2}\ln|x^2-6x+12| + \frac{\sqrt{3}}{3}\,\text{arctg}\left(\frac{x-3}{\sqrt{3}}\right) + c}$$

(5) $\displaystyle\int \frac{1}{x^3+x^2-x-1}\,dx$

$B(x) = x^3+x^2-x-1$

$= x^2(x+1)-(x+1) =$

$= (x+1)(x^2-1) =$

$= (x+1)(x+1)(x-1) =$

$= (x+1)^2(x-1)$

$$\frac{1}{(x+1)^2(x-1)} = \frac{\alpha}{(x+1)} + \frac{\beta}{(x+1)^2} \to \alpha(x+1)^2+\beta(x-1) = 1$$

$$\alpha x^2+2\alpha x+1+\beta x-\beta = 1 \qquad \alpha = 0 \text{ assurdo}$$

$$\frac{1}{(x+1)^2(x-1)} = \frac{\alpha}{(x-1)} + \frac{\beta x+\gamma}{(x+1)^2} \to \alpha(x+1)^2+\beta\cancel{x}+\gamma(x-1) = 1$$

$$\alpha x^2+2\alpha x+\alpha+\beta x^2-\beta x+\gamma x-\gamma = 1$$

$$(\alpha+\beta)x^2+x(2\alpha-\beta+\gamma)+\alpha-\gamma = 1$$

$\alpha+\beta=0$

$2\alpha-\beta+\gamma=0$ } non ci sono termini con x^2 o x

$\alpha-\gamma=1$

$$\begin{cases}\alpha+\beta=0\\2\alpha-\beta+\gamma=0\\\alpha-\gamma=1\end{cases} \to \begin{cases}\alpha=-\beta\\3\alpha+\gamma=0\\\gamma=\alpha-1\end{cases} \qquad \begin{matrix}\alpha=-\beta & \beta=-1/4\\4\alpha=1 & \alpha=+1/4\\\gamma=-3/4 & \end{matrix}$$

$$\frac{1}{(x+1)^2(x-1)} = \frac{1/4}{(x-1)} + \frac{(-1/4)x-3/4}{(x+1)^2}$$

$$\int\frac{1}{(x+1)^2(x-1)}\,dx = \frac{1}{4}\int\frac{1}{x-1}\,dx - \frac{1}{4}\int\frac{x+3}{(x+1)^2}\,dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\int\frac{x+3}{x^2+2x+1}\,dx$$

$f(x) = x^2+2x+1$

$f'(x) = 2x+2$ il numeratore deve essere questo

$$\frac{1}{4}\ln|x-1| - \frac{1}{4}\cdot\frac{1}{2}\cdot\cancel{2}\int\frac{2x+6}{x^2+2x+1}\,dx$$

$$\frac{1}{4}\ln|x-1| - \frac{1}{4}\cdot\frac{1}{2}\int\frac{2x+2}{x^2+2x+1}+\frac{4}{x^2+2x+1}\,dx$$

$$\frac{1}{4}\ln|x-1| - \frac{1}{4}\left(\frac{1}{2}\int\frac{2x+2}{x^2+2x+1}\,dx + 2\int\frac{1}{(x+1)^2}\,dx\right.$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\left[\frac{1}{2}\ln(x^2+2x+1) + 2\,\frac{(x+1)^{-1}}{-1}\right] + c$$

$$\frac{1}{4}\ln|x-1| - \frac{1}{8}\ln(x+1)^2 + \frac{2}{2(x+1)} + c$$

$$\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{2}{2(x+1)} + c$$

(6) $\displaystyle\int \frac{x^4-3x^2+5}{x^2-2x-3}\,dx$

$A(x) = x^4-3x^2+5$ $\quad \partial A \geq \partial B$

$\Delta < 0$

Divisione Tra polinomi

Divido il Termine di grado massimo di sinistra con il Termine di grado massimo di destra (sempre lo stesso)

$$
\begin{array}{rrrrr|l}
x^4 + 0x^3 & -3x^2 & +0x & +5 & & x^2-2x-3 \\
-x^4+2x^3 & +3x^2 & & & & \hline x^2+2x+4 \\
\hline
// \; 2x^3 & +0x^2 & +0x & +5 & & \\
-2x^3 & +4x^2 & -6x & & & \\
\hline
// & 4x^2 & -6x & +5 & & \\
& -4x^2 & +8x & +12 & & \\
\hline
& // & 2x & +17 & &
\end{array}
$$

$$A(x) = (x^2+2x+4)\underbrace{(x^2-2x-3)}_{B(x)} + (14x+17)$$

$$\int \frac{A(x)}{B(x)} = \int x^2+2x+4\,dx + \int \frac{14x+17}{x^2-2x-3}\,dx$$

$$= \frac{x^3}{3} + x^2 + 4x + \int \frac{14x+17}{x^2-2x-3}\,dx$$

$B(x) = x^2-2x-3$

$\Delta > 0$

$(x-3)(x+1)$

$$\frac{14x+17}{(x-3)(x+1)} = \frac{\alpha}{(x-3)} + \frac{\beta}{(x+1)} \rightarrow \alpha(x+1)+\beta(x-3) = 14x+17$$

$$\alpha x + \alpha + \beta x - 3\beta = 14x+17$$

$$\begin{cases} \alpha+\beta = 14 \\ \alpha - 3\beta = 17 \end{cases} \qquad // \; 4\beta = -3 \quad \rightarrow \begin{cases} \beta = -3/4 \\ \alpha = 17 + 3\left(-\frac{3}{4}\right) \end{cases} \rightarrow \begin{cases} \beta = -3/4 \\ \alpha = \frac{59}{4} \end{cases}$$

$$\frac{14x+17}{(x-3)(x+1)} = \frac{59/4}{(x-3)} + \frac{(-3/4)}{x+1}$$

$$\int \frac{14x+17}{(x-3)(x+1)}\,dx = \frac{59}{4}\int \frac{1}{(x-3)}\,dx - \frac{3}{4}\int \frac{1}{x+1}\,dx = \frac{59}{4}\ln|x-3| - \frac{3}{4}\ln|x+1| + c$$

RISULTATO: $\displaystyle \frac{x^3}{3} + x^2 + 4x + \frac{59}{4}\ln|x-3| - \frac{3}{4}\ln|x+1| + c = \int \frac{x^4-3x^2+5}{x^2-2x-3}\,dx$

INTEGRAZIONE PER PARTI

$$\frac{d}{dx} f(x)\, g(x) = \frac{d}{dx} f(x) \cdot g(x) + f(x) \cdot \frac{d}{dx} g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}(f(x)g(x)) - f(x)\frac{d}{dx}g(x) \qquad \int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

Esempio: 1

$$\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\,\frac{1}{\cancel{x}}\, dx \Rightarrow \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$$

$f'(x) = x \quad f(x) = x^2/2$

$g(x) = \ln \quad g'(x) = \frac{1}{x}$

2 $$\int \sin^2 x\, dx = \int \sin x \sin x\, dx \to -\cos x \sin x - \int -\cos x \cos x\, dx$$

$f'(x) = \sin x \quad f(x) = -\cos x$

$g(x) = \sin x \quad g'(x) = \cos x$

$$\to -\cos x \sin x + \int \cos^2 x\, dx = -\cos x \sin x + \int (1 - \sin^2 x)\,dx$$

$$= -\cos x \sin x + \int dx - \int \sin^2 x\, dx = -\cos x \sin x + x - \int \sin^2 x\, dx$$

$$= \int \sin^2 x\, dx = -\cos x \sin x + x - \int \sin^2 x\, dx \to 2\int \sin^2 x\, dx = -\cos x \sin x + x$$

$$\Rightarrow \int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{2}\cos x \sin x + c$$

3 $$\int \cos(\ln(x))\, dx = x\cos(\ln(x)) + \int \cancel{x} \sin(\ln x)\,\frac{1}{\cancel{x}}\, dx \to \cancel{x\cos(\ln(x)) - \cos(\ln x)}$$

$f'(x) = 1 \quad f(x) = x$

$g(x) = \cos(\ln(x)) \to g'(x) = -\sin(\ln(x)) \cdot \frac{1}{x}$

$$x\cos(\ln(x)) + \int \sin(\ln(x))\, dx \to \int \sin(\ln(x))\, dx \quad \text{(PER PARTI)}$$

$f'(x) = 1 \to f(x) = x$

$g(x) = \sin(\ln(x)) \to g'(x) = \cos(\ln(x))\,\frac{1}{x}$

$$x\cos(\ln(x)) + \left(x\sin(\ln(x)) - \int \cancel{x}\cos(\ln(x))\,\frac{1}{\cancel{x}}\, dx\right)$$

$$\int \cos(\ln(x))\, dx = x\cos(\ln(x)) + x\sin(\ln(x)) - \int x\cos(\ln(x))\, dx \to$$

$$2\int \cos(\ln(x))\, dx = x\cos(\ln(x)) + x\sin(\ln(x))$$

$$\int \cos(\ln(x))\, dx = \frac{1}{2}x\left(\cos(\ln(x)) + \sin(\ln(x))\right) + c$$

4) $\int \ln(1+x^2)\,dx = x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \ln(1+x^2) \qquad g' = \frac{1}{1+x^2}\,2x$

$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2}\,dx$

$\int \frac{x^2}{1+x^2}\,dx = \int \frac{x^2+1-1}{x^2+1} = \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx$

$x - \text{arctg}\, x + c$

$= x\ln(1+x^2) - 2x + 2\,\text{arctg}\, x + c$