# POLINOMIO DI TAYLOR

~~Teorema~~ Polinomio di TAYLOR $f: A \subseteq \mathbb{R} \to \mathbb{R}$ funzione

$x_0 \in A$ $f$ derivabile $n$ volte in $U(x_0)$

$$T_{x_0}(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$(n) \to$ derivata $n$-esima

polinomio di TAYLOR centrato in $x_0$ di ordine $n$

Se $x_0 = 0$

$$T_0(x) = f(0) + f'(0)x + \frac{f''(0)}{2!} + \dots + \frac{f^{(n)}(0)}{n!}x^n$$

POLINOMIO DI MAC LAURIN di ordine $n$ (centrato in 0)

## SVILUPPO DI TAYLOR - MAC LAURIN

$$f(x) = T_{x_0}^n(x) + \underbrace{o((x-x_0)^n)}_{\text{resto di Peano}} \to$$

famiglia di funzioni che tendono a 0 più velocemente di $(x-x_0)^n$ per $x \to x_0$.

Esempio $f(x) = \ln(1+x) \to$ Sviluppo di Mac Laurin al 3° ordine $x_0 = 0$

$f(0) = \ln(1) = 0$ $\quad f'(x) = \frac{1}{1+x} \Rightarrow f'(0) = 1$ $\quad f''(x) = \frac{-1}{(1+x)^2} \Rightarrow f''(0) = -1$ $\quad f'''(x) = \frac{2}{(x+1)^3} \Rightarrow f'''(0) = 2$

$$\ln(1+x) = \cancel{f}\, 0 + 1x - \frac{1}{2}x^2 + \frac{2}{3!}x^3 + o(x^3) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + o(x^3)$$

$\to$ termini che vanno a 0 più velocemente di $x^4$ per $x \to 0$

Sviluppo di TAYLOR di ordine 3 centrato in $x_0 = 1$ $\quad f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x}$ $\quad f(1) = \ln(2)$

$f''(x) = \frac{-1}{(x+1)^2}$ $\quad f'(1) = \frac{1}{2}$

$f'''(x) = \frac{2}{(1+x)^3}$ $\quad f''(1) = -\frac{1}{4}$

$f'''(1) = \frac{2}{8} = \frac{1}{4}$

$$\ln(1+x) \underset{x \to 1}{=} \ln(2) + \frac{1}{2}(x-1) - \frac{1/4}{2!}(x-1)^2 + \frac{1/4}{3!}(x-1)^3 + o(x-1)^3$$

$$\ln(2) + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{24}(x-1)^3 + o(x-1)^3$$

Esempio $f(x) = \frac{1}{x-1}$ $\quad$ Sviluppo MAC LAURIN al 3° ordine $x_0 = 0$

$f(0) = -1$

$f'(x) = \frac{-1}{(x-1)^2}$ $\quad f'(0) = -1$

$f''(x) = \frac{2}{(x-1)^3}$ $\quad f''(0) = -2$

$f'''(x) = \frac{-6}{(x-1)^4}$ $\quad f'''(0) = -6$

$$\frac{1}{x-1} = -1 - 1x - \frac{2}{2!}x^2 - \frac{6}{3!}x^3 + o(x^3)$$

$$= -1 - x - x^2 - x^3 + o(x^3)$$

Sviluppo di TAYLOR di 3° ordine centrato in $x_0 = 2$ di $\frac{1}{1-x}$

$f(2) = 1$

$f'(x) = \frac{-1}{(x-1)^2}$ $\quad f'(2) = -1$

$f''(x) = \frac{2}{(x-1)^3}$ $\quad f''(2) = 2$

$f'''(x) = \frac{-6}{(x-1)^4}$ $\quad f'''(2) = -6$

$$\frac{1}{1-x} = 1 - 1(x-2) + \frac{2}{2}(x-2)^2 - \frac{6}{6}(x-2)^3 + o(x-2)^3$$

$$1 - (x-2) + (x-2)^2 - (x-2)^3 + o(x-2)^3$$

Esempio $f(x) = \text{sen}(x^2)$ SVILUPPO DI MAC LAURIN $x_0 = 0$

$f(0) = 0$

$f'(x) = 2x\cos(x^2)$ $\quad f'(0) = 0$

$f''(x) = 2\cos(x^2) - 4x^2 \text{sen}(x^2)$ $\quad f''(0) = 2$

$f'''(x) = -4x \sin x^2 - 8x \text{sen}(x^2) - 8x^3 \cos(x^2)$ $\quad f'''(0) = 0$

$$\underline{\sin x^2} = 0 + 0x + \frac{2}{2}x^2 + 0x^3 + 0x^4 + o(x^4) = \underline{x^2 + o(x^4)}$$

$$\tau = x^2 \quad \sin(\tau) = \tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \frac{\tau^7}{7!} \dots + o(\tau^7)$$

$$\downarrow$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{6} + \frac{(x^2)^5}{5!} - \frac{(x^2)^7}{7!} + o(x^{14}) \Rightarrow \underbrace{x^2}_{\text{grado } 2} - \underbrace{\frac{x^6}{6}}_{\text{grado } 6} + \underbrace{\frac{x^{10}}{5!}}_{\text{grado } 10} - \underbrace{\frac{x^{14}}{7!}}_{\text{grado } 14} + o(x^{14})$$

ESEMPIO $f(x) = [\ln(1+x)]^2$ Sviluppo di mac Laurin $x_0 = 0$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$[\ln(1+x)]^2 = \left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right)^2 = x^2 + \frac{x^4}{4} + \frac{x^6}{9} \overbrace{- x^3 + \frac{2}{3}x^4 - \frac{1}{3}x^5}^{\text{doppi prodotti}} + o(x^6) + \overbrace{2o(x^3)\left(x - \frac{x^2}{2} + \frac{x^3}{3}\right)}^{+o(x^4)+o(x^5)+o(x^6)}$$

$$= x^2 - x^3 + \frac{11}{12}x^4 - \frac{1}{3}x^5 + \frac{x^6}{9} + o(x^6) + o(x^4) + o(x^5)$$

Vanno a 0 più velocemente di

$$= x^2 - x^3 + \frac{11}{12}x^4 + o(x^4)$$

~~Esempio~~ Esempio $f(x) = \cos(e^x - 1)$ sviluppo mac Laurin $x_0 = 0$ $\quad$ per $x \to 0$ $\quad e^x - 1 \to 0$

$$\cos(\tau) = 1 - \frac{\tau^2}{2} + \frac{\tau^4}{4!} + o(\tau^4) \qquad e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3) \text{ per } x \to 0$$

$\underline{\tau = e^x - 1}$ $\quad \tau \to 0$

$$\underbrace{e^x - 1 = x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)}_{\tau}$$

$$\cos(\tau) = 1 - \frac{\tau^2}{2} + \frac{\tau^4}{4!} + o(\tau^4) \Rightarrow 1 - \frac{1}{2}\left(x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)\right)^2 + \frac{1}{24}\left(x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)\right)^4 + o\left(x + \frac{x^2}{2} + \frac{x^3}{6} + \dots\right)^4$$

$$= 1 - \frac{1}{2}\left(x^2 + \frac{x^4}{4} + \frac{x^6}{36} + x^3 + \frac{1}{6}x^5 + \frac{1}{3}x^4 + o(x^6) + o(x^4)\right) +$$

$+\frac{1}{24}(x^4 \dots + o(x^4)) + o(x^4) = 1 - \frac{1}{2}(x^2 + x^3 + \frac{7}{12}x^4) + \frac{1}{24}x^4 + o(x^4)$

li formiamo perché gli altri termini hanno una potenza superiore a $o(x^4)$

$$= \boxed{1 - \frac{1}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{4}x^4 + o(x^4)}$$

$o \to$ non può mai stare da solo al numeratore o denominatore.

ESEMPIO

(1) $\lim_{x \to 1} \frac{\ln(x) - x + 1}{x^2 - 2x + 1} = \lim_{x \to 1} \frac{\ln(x) - (x-1)}{(x-1)^2}$

Cambio variabile $\tau = x - 1$ per $x \to 1$, $\tau \to 0$ $\Rightarrow \lim_{\tau \to 0} \frac{\ln(1+\tau) - \tau}{(\tau)^2} =$

$\ln(1+\tau) = \tau - \frac{\tau^2}{2} + \frac{\tau^3}{3} + o(\tau^3)$ SVILUPPO DI MAC LAURIN
$\tau \to 0$

$\lim_{\tau \to 0} = \frac{\cancel{\tau} - \tau^2/2 + \tau^3/3 + o(\tau^3) - \cancel{\tau}}{\tau^2} = \frac{-\frac{\tau^2}{2} + \frac{t^3}{3} + o(\tau^3)}{\tau^2} \Rightarrow \lim_{\tau \to 0} \frac{\cancel{\tau^2}(-1/2 + \tau/3 + o(\tau))}{\cancel{\tau^2}}$

$= \lim_{\tau \to 0} -1/2 + \tau/3 + o(\tau) = -\frac{1}{2}$

ESERCIZIO $\to$ lo sviluppo deve fermarsi quando almeno a quando numeratore e denominatore hanno lo stesso grado

$\lim_{x \to 0} \frac{e^{2x} - 1 - \sin(2x) - 2x^2}{8x^3}$

- $e^\tau = 1 + \tau + \frac{\tau^2}{2} + \frac{\tau^3}{6} + o(t^3)$
- $\sin(\tau) = \tau - \frac{t^3}{6} + o(\tau^3)$
- $e^{2x} = 2x + 1 + \frac{4x^2}{2} + \frac{8x^3}{6} + o(x^3)$
- $\sin(2x) = 2x - \frac{8x^3}{6} + o(x^3)$

$\lim_{x \to 0} \frac{\cancel{1} + \cancel{2x} + \cancel{2x^2} + 4/3x^3 + o(x^3) - \cancel{1} - \cancel{2x} + 4/3x^3 - \cancel{2x^2}}{8x^3} = \lim_{x \to 0} \frac{8/3x^3 + o(x^3)}{8x^3} \Rightarrow \lim_{x \to 0} \frac{x^3}{x^3} \frac{8/3 + o(1)}{8} =$

ESERCIZIO $\lim_{x\to 0} \frac{\sin(x) - sh(x)}{x^3}$

$\sin(x) = x - \frac{x^3}{6} + o(x^3)$

$sh(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}(e^x - e^{-x}) =$

$= \frac{1}{2}\Big(\underbrace{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)}_{e^x}\Big) - \Big(\underbrace{1 - x + \frac{x^2}{2} - \frac{x^3}{6} + o(x^3)}_{e^{-x}}\Big)$

$= \frac{1}{2}\Big[\cancel{1} + x + \cancel{\frac{x^2}{2}} + \frac{x^3}{6} - \cancel{1} + x - \cancel{\frac{x^2}{2}} + \frac{x^3}{6} + o(x^3)\Big)$

$= \frac{1}{2}\Big(2x + \frac{2}{6}x^3 + o(x^3)\Big) = x + \frac{1}{6}x^3 + o(x^3)$

$\lim_{x\to 0} \frac{\cancel{x} - x^3/6 + o(x^3) - (\cancel{x} + \frac{1}{6}x^3 + o(x^3))}{x^3}$

$\lim_{x\to 0} \frac{-1/3\, x^3 + o(x^3)}{x^3} = \lim_{x\to 0} \frac{x^3(-1/3 + o(1))}{x^3\ (1)} =$

$= -\frac{1}{3}$

ESERCIZIO $\lim_{x\to 0} \frac{e^x - xe^x - 1}{x^2(1-\cos x)} = \lim_{x\to 0} \frac{e^x(1-x) - 1}{x^2(1-\cos x)} =$

$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$

$\cos(x) = 1 - \frac{1}{2}x^2 + o(x^2)$

$\Rightarrow$ Ci fermiamo all'ordine due perché il grado del denominatore è $2 \Rightarrow x^{2}$

no, perché è superiore al grado di o

$= \lim_{x\to 0} \frac{(1-x)(1+x+x^2/2+o(x^2)) - 1}{x^2(\cancel{1} - 1 + \frac{1}{2}x^2 + o(x^2))} = \frac{\cancel{1} + \cancel{x} + x^2/2 + o(x^2) - \cancel{x} - x^2 - x^3/2 + o(x^3) - \cancel{1}}{\frac{1}{2}x^4 + o(x^4)}$

$\lim_{x\to 0} \frac{-1/2\, x^2 + o(x^2)}{\frac{1}{2}x^4 + o(x^4)} = \lim_{x\to 0} \frac{\cancel{x^2}(-1/2 + o(1))}{\cancel{x^2}(\frac{1}{2}x^2 + o(x^2))} = \lim_{x\to 0} \frac{-1/2 + o(1)}{0} \Rightarrow -\infty$

-Non prendiamo in considerazione i termini che sono di grado superiore all' o di grado più basso. Se l' $o(x^2)$, non prendiamo $x^3$

ESERCIZIO

$\lim_{x\to +\infty} x - \underbrace{x^2 \ln\left(1 + \sin\left(\frac{1}{x}\right)\right)}_{g(x)} \Rightarrow \lim_{x\to+\infty} x^2 \ln\left(1 + \sin\left(\frac{1}{x}\right)\right)$ cambio variabile $\frac{1}{x} = t$ per $x \to +\infty$, $t \to 0^+$

$= \lim_{t\to 0^+} \frac{1}{t^2} \ln(1 + \sin(t))$ $\qquad \sin t = t - \frac{t^3}{3!} + o(t^3)$ $\qquad \ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} + o(t^3)$

$= \ln(1 + \sin(t)) \Rightarrow \ln\Big(1 + \underbrace{\Big(t - \frac{t^3}{6} + o(t^3)\Big)}_{\text{diventa la } t \text{ di } \ln}\Big) = t - \frac{t^3}{6} + o(t^3) - \frac{1}{2}\Big(t - \frac{t^3}{6} + o(t^3)\Big)^2 + \frac{1}{3}\Big(t - \frac{t^3}{6} + o(t^3)\Big)^3$

$= t - \frac{t^3}{6} - \frac{1}{2}\Big(t^2 - \underbrace{\frac{1}{3}t^4 + \frac{t^6}{36}}_{\text{trascurabili perché c'è } o(t^3)} + o(t^4)\Big) + \frac{1}{3}\Big(t^3 \dots$ gli altri termini sono maggiori dell'ordine di o, non li scriviamo$\Big) + o(t^3)$

$= t - \frac{1}{2}t^2 - \frac{t^3}{6} + \frac{1}{3}t^3 + o(t^3) \Rightarrow t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + o(t^3)$

$\lim_{t\to 0^+} \frac{1}{t^2}\Big(t - \frac{t^2}{2} + \frac{t^3}{6} + o(t^3)\Big) \Rightarrow \lim_{t\to 0^+} \frac{1}{t} - \frac{1}{2} + \frac{1}{6}t + o(t) = \infty$

$$\lim_{x \to \infty} x - x^2 \ln\left(1 + \sin\left(\frac{1}{x}\right)\right) \qquad \infty - \infty \quad \text{F.I}$$

$$\lim_{t \to 0} \cancel{\frac{1}{t}} - \left[\cancel{\frac{1}{t}} - \frac{1}{2} + \frac{1}{6} t + o(t)\right] = \frac{1}{2}$$

ESERCIZIO Determinare gli asintoti obliqui di $f(x) = x^3 \operatorname{arctg}\left(\frac{1}{x}\right) - x^2 + x$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\cancel{x}\, x^2 \operatorname{arctg}(1/x) - x + 1}{\cancel{x}} = \lim_{x \to +\infty} x^2 \operatorname{arctg}\left(\frac{1}{x}\right) - x + 1$$

Cambio variabile $\frac{1}{x} = t$ per $\begin{matrix} x \to +\infty \\ t \to 0^+ \end{matrix}$ $= \lim_{t \to 0^+} \frac{1}{t^2} \underline{\operatorname{arctg}(t)} - \frac{1}{t} + 1$

$$\operatorname{arctg}(t) = t - \frac{t^3}{3} + \frac{t^5}{5} + o(t^5) \Rightarrow \lim_{t \to 0^+} \frac{1}{t^2}\left(t - \frac{t^3}{3} + o(t^3)\right) - \frac{1}{t} + 1 =$$

$$= \lim_{t \to 0^+} \cancel{\frac{1}{t}} - \frac{t}{3} + o(t) - \cancel{\frac{1}{t}} + 1 = 1 \qquad m = 1$$

$$q = \lim_{x \to +\infty} f(x) - mx \Rightarrow \lim_{x \to +\infty} x^3 \operatorname{arctg}\left(\frac{1}{x}\right) - x^2 + \cancel{x} - \cancel{x} \Rightarrow \lim_{x \to +\infty} x^3 \operatorname{arctg}\left(\frac{1}{x}\right) - x^2$$

Cambio variabile $\frac{1}{x} = t$ per $\begin{matrix} x \to +\infty \\ t \to 0^+ \end{matrix}$ $= \lim_{t \to 0^+} \frac{1}{t^3} \operatorname{arctg}(t) - \frac{1}{t^2} =$

$$= \lim_{t \to 0^+} \frac{1}{t^2}\left(\frac{1}{t} \operatorname{arctg}(t) - 1\right) \Rightarrow \lim_{t \to 0^+} \frac{1}{t^2}\left(\frac{1}{t}\left(t - \frac{t^3}{3} + o(t^3)\right) - 1\right) =$$

$$\lim_{t \to 0^+} \frac{1}{t^2}\left(\cancel{1} - \frac{t^2}{3} + o(t^2) - \cancel{1}\right) = \lim_{t \to 0^+} -\frac{1}{3} + o(1) = -\frac{1}{3} \qquad q = -\frac{1}{3}$$

$$y = mx + q \Rightarrow y = x - \frac{1}{3}$$

ESERCIZIO $\lim_{x\to 0} \frac{5^{1+tg^2(x)}-5}{1-\cos(x)} = \lim_{x\to 0} \frac{5\left(5^{tg^2(x)}-1\right)}{1-\cos(x)}$

$5^{tg^2(x)} = e^{\ln 5^{tg^2(x)}} = e^{tg^2(x)\ln(5)}$

$e^t = 1+t+\frac{t^2}{2}+o(t^2)$

$tg(u) = u+\frac{u^3}{3}+o(u^3)$

$$e^{tg^2(x)\ln(5)} = 1+\ln(5)\,tg^2(x)+o(\ln(x)\,tg^2(x))$$
$$= 1+\ln(5)\left(x+\frac{x^3}{3}+o(x^3)\right)^2+o(x^2)$$
$$= 1+\ln(5)\,x^2+o(x^2)$$

$$\lim_{x\to 0}\frac{5\left(5^{tg^2(x)}-1\right)}{1-\cos(x)} = \lim_{x\to 0}\frac{5(\cancel{1}+\ln(5)x^2+o(x^2)-\cancel{1})}{\cancel{1}-\cancel{1}+\frac{1}{2}x^2+o(x^2)} = \lim_{x\to 0}\frac{5\ln(5)x^2+o(x^2)}{\frac{1}{2}x^2+o(x^2)} =$$

$= 10\ln(5)$

ESERCIZIO $\lim_{x\to+\infty}\frac{\sqrt[3]{2x^2+x^3}-xe^{2/3x}}{arctg\left(\frac{1}{x^2}\right)} = \lim_{x\to+\infty}\frac{\sqrt[3]{x^3\left(1+\frac{2}{x}\right)}-xe^{\frac{2}{3x}}}{arctg\,\frac{1}{x^2}}$

Cambio variabile $\frac{1}{x}=t$ per $x\to+\infty$ $t\to 0^+$

$$=\lim_{t\to 0^+}\frac{\frac{1}{t}\sqrt[3]{(1+2t)}-\frac{1}{t}e^{2t/3}}{arctg(t^2)} = \lim_{t\to 0^+}\frac{\frac{1}{t}(1+2t)^{1/3}-e^{2t/3}}{arctg(t^2)} =$$

$(1+u)^\alpha = 1+\alpha u+\frac{\alpha(\alpha-1)}{2}u^2+\frac{\alpha(\alpha-1)(\alpha-2)}{\cancel{6}\,3!}u^3+o(u^3)$, $u\to 0$

$arctg(u) = u-\frac{u^3}{3}+\frac{u^5}{5}+o(u^5)$

$e^u = 1+u+\frac{u^2}{2}+\frac{u^3}{6}+o(u^3)$

$$=\lim_{t\to 0^+}\frac{1}{t}\left[1+\frac{1}{3}(2t)+\frac{1}{2}\frac{1}{3}\left(\frac{1}{3}-1\right)(2t)^2+o(t^2)-\left(1+\frac{2}{3}t+\frac{1}{2}\frac{4}{9}t^2+o(t^2)\right)\right]$$

$$=\cancel{1}+\cancel{\frac{2}{3}t}-\frac{4}{9}t^2-\cancel{1}-\cancel{\frac{2}{3}t}-\frac{2}{9}t^2+o(t^2) = -\frac{6}{9}t^2+o(t^2) = -\frac{2}{3}t^2+o(t^2)$$

$$=\lim_{t\to 0^+}\frac{\frac{1}{t}\left(-2/3\,t^2+o(t^2)\right)}{t^2+o(t^2)} = \lim_{t\to 0}\frac{1}{t}\left(-2/3+o(1)\right) = -\infty$$

Esercizio $f(x) = xe^x$ mostrare che $f$ è localmente invertibile in $x_0 = 0$
Scrivere la formula di MacLaurin della funzione inversa $g(y)$

Calcolare $\lim_{y \to 0} \frac{g(y) - y - \ln(1+y^2)}{y^2}$

$f$ è localmente invertibile in $U(0)$ continua e iniettiva (monotona) $Df = \mathbb{R}$

$f' = e^x + xe^x = e^x(1+x)$ $f'(x) > 0$ per $1+x > 0$ $x > -1$

$f$ è monotona crescente in $U(0)$ (localmente invertibile)

$g'(y) = \frac{1}{f'(x)}$ $x_0 = 0$ $f(0) = 0 \to y_0 = 0$

$f'(0) = e^0(1+0) = 1$ $g'(0) = \frac{1}{f'(0)} = \frac{1}{1}$

$g''(y) = \frac{-f''(x)}{(f'(x))^2}$ $f''(x) = e^x(1+x) + e^x = e^x(2+x)$

$f''(0) = e^0(2+0) = 2$

$g''(0) = \frac{-f''(0)}{(f'(0))^2} = \frac{-2}{1} = -2$

$g'''(y) = \frac{-f'''(x) \cdot (f'(x))^2 + f''(x) \cdot 2f'(x) \cdot f''(x)}{(f'(x))^4}$ $f'''(x) = e^x(3+x)$
$f'''(0) = e^0(3+0) = 3$

$g'''(y) = \frac{-3 \cdot 1^2 + 2 \cdot 2 \cdot 2}{1^4} \to 5$

$g(y) = g(0) + g'(0) \cdot y + \frac{g''(0)}{2!} \cdot y^2 + \frac{g'''(0)}{3!} \cdot y^3 + o(y^3) = 0 + 1 \cdot y - \frac{2}{2} y^2 + \frac{5}{6} y^3 + o(y^3)$

$= y - y^2 + \frac{5}{6} y^3 + o(y^3)$

$\lim_{y \to 0} \frac{g(y) - y - \ln(1+y^2)}{y^2} \Rightarrow \frac{\cancel{y} - y^2 + \frac{5}{6}y^3 + o(y^3) - \cancel{y} - (y^2 - \frac{y^4}{2} + o(y^4))}{y^2} = \frac{-2y^2 + \frac{5}{6}y^3 + o(y^3)}{y^2}$

$= \lim_{y \to 0} = -2$