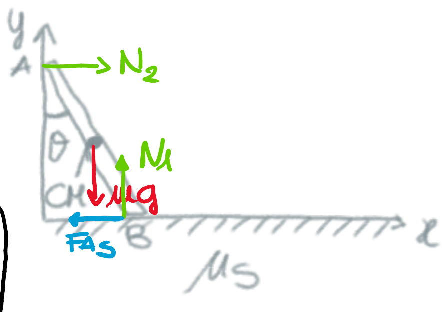
24/11/2020

martedì 24 novembre 2020 15:00

RIPASSO:
- MOMENTO d'inerzia
- equazioni cardinali

1



$\theta = ?$

bilancio delle forze totali
sull'asse x:
$F_{TOT}^x = 0$

sull'asse y:
$F_{TOT}^y = 0$

equazione dei momenti
$\vec{M}_B = 0$

} le metto a sistema

$$\begin{cases} F_{TOT}^x = 0 \\ F_{TOT}^y = 0 \\ \vec{M}_B = 0 \end{cases} \Rightarrow \begin{cases} N_2 - F_{AS} = 0 \\ N_1 - mg = 0 \\ mg\frac{\ell}{2}\sin\theta - N_2\,\ell\cos\theta = 0 \end{cases}$$

↳ N_1 e $F_{A,S}$ hanno momento pari a zero perché sono applicate nel punto in cui voglio calcolare il momento

$$mg\frac{\ell}{2}\sin\theta - F_{AS}\,\ell\cos\theta = 0$$

$$F_{AS} = \frac{mg\,\ell/2\,\sin\theta}{\ell\cos\theta} = \frac{mg}{2}\,tg\theta$$

$$F_{Amax} = N\mu_s$$

$$\frac{mg}{2}\,tg\theta \le N_1\mu_s \rightsquigarrow mg\,\mu_s$$

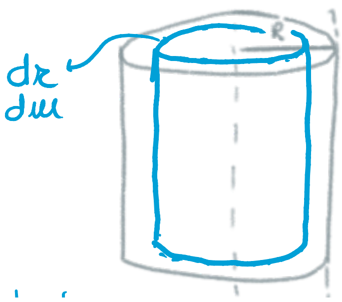
↳ ricavato nel sistema

$$\theta \le arctg\,2\mu_s$$

2

$$I_z = \int_V r^2\,dm$$

$$\rho = \frac{M}{V_c} = \frac{M}{\pi R^2 h}$$

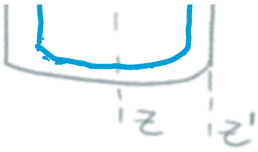


$\rho = \frac{m}{V}$

$dm = \rho\,dV =$

$= \rho\,2\pi r\,dr\,h$

$V_c \quad \pi R^2 h$



$= \rho 2\pi r\, dr\, h$

$$I_z = \int_V r^2 dm = \int r^2\, \rho 2\pi r h\, dr$$

$$I_z = \int_0^R r^2 \frac{M}{\cancel{\pi} R^2 \cancel{h}} 2\cancel{\pi} r \cancel{h}\, dr = \frac{2M}{R^2}\int_0^R r^3 dr = \frac{2M}{R^2}\cdot\frac{R^4}{4} = \frac{MR^2}{2}$$

TEOREMA degli ASSI PARALLELI

$$I_{z'} = I_z + M\cdot R^2$$

↳ distanza dei due assi al quadrato

$$= \frac{MR^2}{2} + MR^2 = 3MR^2$$

caso del cilindro cavo

$$I_z = \int r^2 dm =$$

↳ posso tirarlo fuori perché non dipende da m

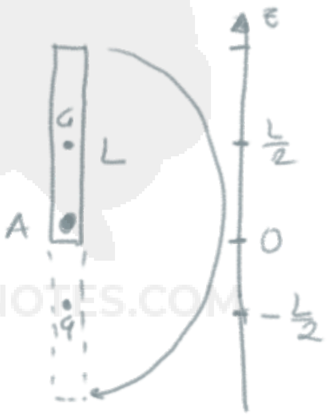
$$= R^2\int dm = R^2 M$$

$$I_{z'} = 2MR^2$$

3

$$E_M = \frac{1}{2}M v_B^2 + \frac{1}{2}I\omega^2 + Mg\frac{L}{2}$$

$\downarrow E_K^{TRASL}$ $\quad \downarrow E_K^{ROT.}$ $\quad \downarrow U$



$$E_M^i = Mg\frac{L}{2}$$

$$E_M^f = -Mg\frac{L}{2} + \frac{1}{2}I_A\omega^2$$

$$E_M^i = E_M^f \qquad I_A = \frac{1}{3}ML^2$$

$$Mg\frac{L}{2} = \frac{1}{2}\cdot\frac{1}{3}ML^2\omega^2 - Mg\frac{L}{2}$$

$$\frac{1}{6}\cancel{M}L^{\cancel{2}}\omega^2 = \cancel{M}g\cancel{L}$$

$$\omega = \sqrt{\frac{6g}{L}}$$

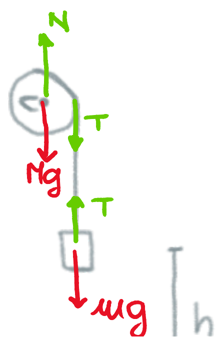
4

Equazione del moto di m

$$mg - T = ma \longrightarrow mg - T = m\alpha R$$

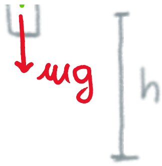
Equazione momenti di M

$$TR = I_c\alpha$$



$$T R = I_c \alpha$$
$$T = mg - m\alpha R$$

$$(mg - m\alpha R)R = I_c\alpha$$
$$mgR - m\alpha R^2 = I_c \alpha \qquad I_c = \frac{1}{2} M R^2$$



$$\alpha = \frac{mgR}{mR^2 + I_c}$$

$$\begin{cases} y(t) = \frac{1}{2} a t^2 \\ y(t_0) = h \end{cases} \longrightarrow t_0 = \sqrt{\frac{2h}{\alpha R}}$$

$$\omega_0 = \alpha t_0 = \alpha \sqrt{\frac{2h}{\alpha R}} = \sqrt{\frac{2h\alpha}{R}} = \sqrt{\frac{2mgh}{mR^2 + I_c}} = 6{,}7 \ rad/s$$

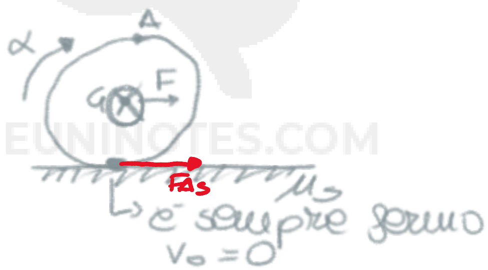
Si poteva fare anche con la conservazione dell'energia

$$E_M^i = 0$$
$$E_M^f = \frac{1}{2} m v_m^2 + \frac{1}{2} I_c \omega^2 - mgh$$
$$\hookrightarrow \omega_0^2 R^2$$

$$E_M^i = E_M^f$$

5

$$V_G = \omega R$$
$$V_A = 2\omega R$$



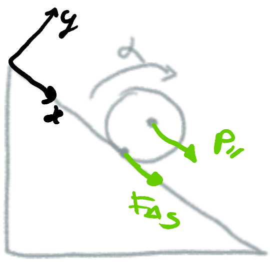
$$\begin{cases} F_0 + F_{AS} = ma \\ M_0 - F_{AS} R = I_{CM}^G \alpha \end{cases}$$

$$\begin{cases} F_{AS} = ma - F_0 \\ M_0 - (ma - F_0)R = I_{CM}^G \alpha \end{cases}$$
$$M_0 - maR + F_0 R = I_{CM}^G \alpha$$
$$M_0 - m\alpha R^2 + F_0 R = I_{CM}^G \alpha$$
$$\alpha = \frac{M_0 + F_0 R}{I_{CM}^G + mR^2} = 26{,}7 \ rad/s^2$$

$$F_{AS} = m\alpha R - F_0 = 3{,}3 \ N$$
$$\mu_s \geq \frac{F_{AS}}{mg} \geq 0{,}3$$

mg

6



$$\begin{cases} x \rightarrow Mg\sin\theta + F_A = Ma_G \\ y \rightarrow N - mg\cos\theta = 0 \\ M^G_{tot} \rightarrow -F_A R = I_G \alpha \end{cases}$$

$$I_G = \frac{1}{2}MR^2$$

$$F_{A,S} = -\alpha \frac{1}{2}\frac{MR^2}{R} = -\frac{\alpha}{2}MR$$

$$Mg\sin\theta - \frac{\alpha}{2}MR = M\alpha R$$

$$\alpha = \frac{2}{3}\frac{g\sin\theta}{R}$$

$$F_{AS} = -\frac{Mg\sin\theta}{3}$$

$$F_{AS} \leq \mu_s N = \mu_s M g\cos\theta$$

$$\theta \leq \arctan(3\mu_s)$$

7

$$\cancel{\rho_{H_2O}} g(\Delta h + d + \cancel{h_0}) = \cancel{\rho_{H_2O}} g \cancel{h_0} + \rho_e g d$$

$$\rho_{H_2O} g\Delta h + \rho_{H_2O} g d = \rho_e g d$$

$$\rho_e = \frac{\rho_{H_2O}(\Delta h + d)}{d}$$