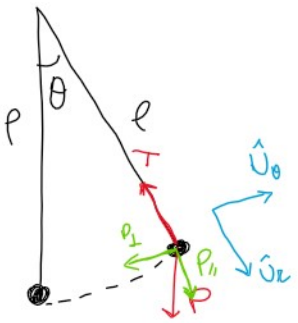
03/11/2020

martedì 3 novembre 2020 08:09

RIPASSO

MOTO DEL PENDOLO



$\vec{F} = m\vec{a}$

$-(r) \to T - mg\cos\theta = m\frac{v^2}{l}$

$(t) \to -mg\sin\theta = m a_T$

$a_T = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2} = r\alpha$

$v = \omega r$

$v = \frac{d\theta}{dt} r$

$-mg\sin\theta = ml\frac{d^2\theta}{dt^2}$

$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$

$\sin\theta \simeq \theta + \dots \quad \leftarrow$ M-L

$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$

identica a quella del moto armonico

$\theta(t) = A\cos(\omega_0 t + \varphi_0)$

$\hookrightarrow$ NON È LA DERIVATA DI θ

$\omega = \frac{d\theta}{dt} = -A\omega_0\sin(\omega_0 t + \varphi_0)$

$\alpha = \frac{d''\theta}{dt^2} = -A\omega_0^2\cos(\omega_0 t + \varphi_0)$

$-A\omega_0^2\cos(\omega_0 + \varphi_0) + \frac{g}{l}A\cos(\omega_0 t + \varphi_0) = 0$

se $\theta(t) = A\cos(\omega_0 t + \varphi_0)$ è soluzione
$\Rightarrow$ l'eq. è soddisfatta $\forall t$

$A\cos(\omega_0 t + \varphi_0)\left[-\omega_0^2 + \frac{g}{l}\right] = 0 \Rightarrow \boxed{\omega_0^2 = \frac{g}{l}}$

$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$

$\omega_0 = \sqrt{\frac{g}{l}}$

$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$

$\frac{d\theta}{dt} = -A\omega_0\sin(\omega_0 t + \varphi_0)$

A e φ_0 sono costanti e si scelgono in base alle condizioni iniziali

- $\theta(0) = \theta_0$
- $v(0) = 0 \to \omega \frac{d\theta}{}(0) = 0$

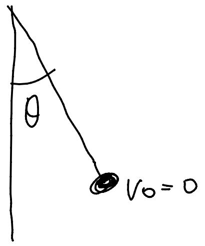
in base alle condizioni iniziali

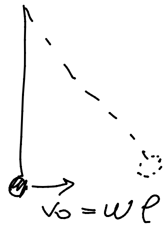
- $\theta(0) = \theta_0$
- $v(0) = 0 \longrightarrow \omega \frac{d\theta}{dt}(0) = 0$

$$\begin{cases} \theta(0) = A\cos\varphi_0 = \theta_0 \\ \frac{d\theta}{dt} = \omega(0) = -A\omega_0 \sin\varphi_0 = 0 \end{cases} \rightarrow \begin{cases} A = \theta_0 \\ \varphi_0 = 0 \end{cases}$$

Fissate le costanti le sostituisco nella soluzione

$$\theta(t) = \theta_0 \cos(\omega_0 t)$$

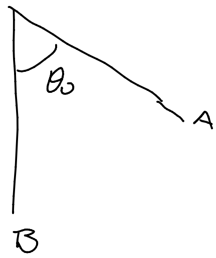




$\theta = \theta_0 \cos(\omega t)$ $\qquad$ $\theta = \theta_0 \sin(\omega t)$

es

$\ell = 1\,m$
$m = 2\,kg$
$\theta_0 = \frac{\pi}{6}$ $\qquad \omega_0 = \sqrt{\frac{g}{\ell}} = 3,13\ rad/s$



$V_B = ?$

① legge oraria
② conservazione dell'energia

① $v = \omega\ell = \frac{d\theta}{dt}\ell$ $\qquad \theta = \frac{\pi}{6}\cos(\omega_0 t)$

$$\frac{d\theta}{dt} = -\frac{\pi}{6}\omega_0 \sin(\omega_0 t)$$

nel punto più basso $\omega = -\frac{\pi}{6}\omega_0$

$\omega_0 = \frac{2\pi}{T}$ $\qquad t = \frac{T}{4}$ $\qquad \omega_0 t = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$

$V_B = \frac{\pi}{6}\omega_0 \ell \simeq 1,64\ m/s$

↳ approssimato perchè $\sin\theta \simeq \theta$

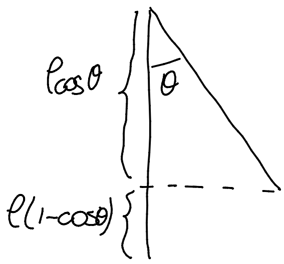
②

$E_{TOT\,A} = E_{TOT\,B}$

$mg\ell(1-\cos\theta) = \frac{1}{2} m V_B^2$

$V_B = \sqrt{2g\ell\left(1-\frac{\sqrt{3}}{2}\right)} = 1,62\ m/s$

↳ esatto



T in funzione di θ

$$T - mg\cos\theta = m\frac{v^2(\theta)}{\ell}$$

$$mg\ell(1-\cos\theta_0) = mg\ell(1-\cos\theta) + \frac{1}{2}mv^2$$

$$v^2 = 2g\ell(\cos\theta - \cos\theta_0)$$

$$T = mg\cos\theta + m2g(\cos\theta - \cos\theta_0)$$

$$v^2 = 2g\ell(\cos\theta - \cos\theta_0)$$

$$T = mg\cos\theta + m2g(\cos\theta - \cos\theta_0)$$

$$T(\theta) = mg\left(3\cos\theta - 2\cos\theta_0\right)$$

$$T_B(0) = mg(3 - 2\cos\theta_0)$$

$\theta_{max} = ?$

$$\frac{1}{2}mv_B^2 = mg\ell(1 - \cos\theta_{max})$$

$$\cos\theta_{max} = -\frac{v_B^2}{2g\ell} + 1$$

$$\theta_{max} = \cos^{-1}\left(1 - \frac{v_B^2}{2g\ell}\right)$$

$$\omega_0 = \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} \neq W = \frac{d\theta}{dt}$$